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A STATISTICAL STUDY OF PSEUDO-CHROMESTHE-SIA AND OF MENTAL-FORMS.

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In the spring of 1892, the Wellesley College class in experimental psychology began a study of cases of pseudo-chromesthesia and of mental "forms." A canvass of the college was undertaken and records of outside cases were also collected. Since that time a few new records have been added and a very detailed investigation of all cases has been made, on the basis of a series of questions which were formulated after the careful study of the first records. The results are summarized here with the briefest possible comment.

In the first table, reference is made only to the records from members of Wellesley College; and so large a number of persons have been consulted that the per cents may perhaps fairly be supposed to suggest the common prevalence of the phenomenon.²

² Since the completion of this paper, a canvass has been made of the students who entered Wellesley in the fall of 1892. Its results differ remarkably from those of Summary I., in the far larger proportion of cases, both of pseudo-chromesthesia and of forms.

Of 203 consulted, the number of persons with pseudo-chromesthesia is $\begin{array}{ccc} 32 & (=15.7\%) \\ \text{Number of persons with forms is} & 61 & (=30.2\%) \\ \text{Number of persons with both is} & 17 & (=8.4\%) \end{array}$

The result may be accidental, but it is possible, on the other hand, that among the two hundred or more, last year, whom our questions did not reach, was a relatively large proportion of subjects. It is proposed to attempt a mediation between the two results, by continuing this canvass with successive freshman classes.

¹A short account of this work was given in the AMERICAN JOURNAL OF PSYCHOLOGY, Vol. V., No. 2. Variations of the present record from that are due chiefly to the new material collected; but occasionally to later, more detailed statements of the same subjects. The figures given, throughout this article, have been carefully verified, and every effort has been made to interpret accurately, through personal interviews, or through correspondence, the exact meaning of the subjects. Hundreds of letters have been written and scores of interviews have been held.

SUMMARY I.

Comparative Frequency of Pseudo-chromesthesia and of Forms.

Total number of persons consulted,	525
Number of persons with pseudo-chromesthesia,	35 (=6.66%)
Number of persons with forms,	$65 \ (=12.38\%)$
(Note: Number of persons with both,	18=3.42%)

The other summaries deal with all recorded cases, including those outside of Wellesley. The first of these attempts a sub-classification of

Summary II.

Varieties of Forms. (Total Number of Subjects, 85.1)

	Verified.	Constant.	Unverified.	Total.
Month-forms.	55	13	5	73
Number-forms.	50	10	7	67
Day-of-week-forms.	23	23	4	50
Century-forms.	4	7	1	12
Hour-forms.	3	1	1	5
Alphabet-forms.	6	37	2	45
Totals.	141	91	20	252

In the first column of this summary are included cases in which the forms have been drawn in the same way after an interval (in most cases a year, occasionally only a few months) following the first record. A few cases are counted in which the second form varies slightly from the first (for instance, bends at the same number and at the same angle, but to the left rather than to the right). The second column contains the record of forms which did not appear on the first record, where the omission was a mere neglect and the subject explicitly testifies to the constant possession of the forms. The third column includes not only the unverified cases, but also those in which the second drawing of the form differs from the first.

Among the hour-forms and the century-forms are included only the cases in which these are unlike the number-forms; to most subjects the number-form is used for both series. The alphabet-forms are probably, most of them, mere visual

¹ Evidently the record of the same subject appears often under several of these heads.

reproductions of the primer page, so that they hardly

belong to our summarv.1

A few curious forms are not included at all. Among these are forms for piano notes (squares), with lines for violin notes; and an interesting prayer-form, well remembered from the time when the progress from one prayer to another was always the passage from one part into another of the form.2

The next table is a classification of

SUMMARY III.

Varieties of Pseudo-chromesthesia. (Total Number of S	Subjects, 45.3)
Color with letters: With consonants only, With vowels only, With both vowels and consonants,	54 44 20
	29 cases.
Color with the numerals:	10 cases.
With common terms only, With proper names only, Names of people only, Names of days only, Names of months and days, Names of people, months and days,	20 ⁵ 1
Color with music:	38 cases. 8 3 4 8
	23 cases.
Total varieties of pseudo-chromesthesia,	100 ³ cases.

The connection of particular colors with the different letters is a widely varying one. All the associations seem fortuitous, except possibly that of i with black and of o with Dr. Jordan⁶ explains this as certainly due to the white.

¹But there are some distinctive alphabet-forms. Cf. Fig. 3, Plate I.

²See also Fig. 7, Plate I. ³Evidently the record of the same subject often appears under

several of these heads.

This contradiction of Galton's generalization has already been noticed. American Journal of Psychology, Vol. V., No. 2.

There are four cases of color with all words.

Cf. D. S. Jordan, Pop. Sci. Mo. XXXIX. 67.

appearance of the letters; but, in the case of o, an explanation given in one of my records seems equally plausible, and is interesting because the subject is now blind. ""o," she says, "eipher=blank=sheet of white paper."

SUMMARY IV.

Colors with Different Letters.

I.	With i:		
	i is black in	11	cases.
	i is "nearly black" in	4	"
	i is grey in i is "white" or "light" in	3 2	"
	i is cardinal in	1	case.
	Total cases of color-associations with i ,	21	
II.	With o:		
	o is white in	11	cases.
	o is "greyish-white" in 3		
	o is "bluish-white" in 1 o is "colorless or white" in 1 hearly white in	7	"
	o is "white or yellow" in 1	•	
	o is "light" in 1		
	o is grey in	1	case.
	o is "golden-brown" in	1	"
	o is black or "very dark" in	- Z	cases.
	Total cases of color-associations with o,	22	
III.	With a:		
	a is blue in	7	cases.
	a is red in a is of other color in	4 11	"
	a is of other color in		
	Total cases of color-association with a ,	22	
IV.	With e:		
	e is yellow in		cases.
	e is of other color in	15	"
	Total cases of color-associations with e,	23	-
V.	With 8:		
γ.	s is yellow in	6	cases.
	s is red in	3	""
	s is "red or yellow" in	1	case.
	s is "yellowish-red" and "reddish-yellow" in		cases.
	s is of other color (blue 4 times) in	10	
	Total cases of color-association with s,	22	}

The relative frequency of the connection of the color with the sound or with the appearance of letter or word has been carefully studied. Of course in the cases of merely musical association—except in the few of color with the printed notes

¹One of the two records from students of Perkins Institute.

—the color follows the sound; and probably this is also true where merely the vowels or merely the consonants have color, since the distinction here is one of sound, not of appearance. On the other hand, in cases of association with numerals (as distinguished from the names of the numbers), the color follows the appearance. In the remaining varieties, we find the association of color with sound most frequent; so that the name "colored-hearing" is partly justified.

SUMMARY V. Connection of Color with Sound and with Appearance.

a. Color when Letter or Word is					b. Color	WHEN	LETT	ER OR	Wor	is Im	AGINED AS	
Heard (only).	Seen only. Seen only. Both. Total. Unv. Cases.				Total. Cases.	Heard.	Seen.	Both.	Total.	Never.	Unv.	Total. Cases.
14	1	28	43	2	45	11 1 27 39 4 2 45						

To discover exactly the manner in which the color appears to a subject is very difficult. To some the color is so indefinite that it is almost impossible for them to describe it; but to a larger number, the experience is so clear that they assume its universality and can hardly be prevailed upon to describe it in detail. "It was not recorded last year," one subject says of her month-form, "because I did not realize that so simple an arrangement could be a 'form." Great care has, therefore, been exercised to make these results correct interpretations of actual experience.

In the following summary, column (a) includes cases in which the color appears as a background to letter or to word; (b) those in which each letter is colored (as if printed in colored ink); (c) those in which the letters are colored, but of one color; (d) that larger number of instances in which the color appears in more or less vague form—not that of the letters—either after or with word, music¹ or letters: evidently this class contains all cases of musical color-association.

¹In one of this year's records the music-color is thus described: "I imagine a prismatic band somewhere around the keys (I can't decide whether it is over or under them)."

SUMMARY VI.
Manner of Word-color.

(a)	(b)	(c)	(d)	(e)	Total.	Unv.	Total Cases.
1	4	9	19	3	36	2	38

The connection in cases of letter and word-association between the coloring of words and that of their letters, is peculiarly baffling. It is impossible to reduce it to rule; often, in spite of definite associations of color with the letters. the word has a color different from that of any of the letters composing it. Very often such a word, with independent color of its own, may be made to assume the color of its various letters by mentally fixating each of these; but this is ordinarily at the expense of any appreciation of the word as a whole. In the next summary, numbers under (a) refer to cases in which the word follows the color of its initial letter; under (b) and (c) to those in which the color seems to be that of a predominating vowel or letter; under (d) to those in which each letter retains its color. The class of associations in which no connection is traced between letter and word-coloring is marked by (e). It will be observed that, followed closely by this class of the undetermined, the one in which word-coloring follows the initial is best filled. Since the word-colors of the same subject may be determined at different times by different principles, the same "case" may be recorded under more than one of the headings. These double records are indicated in the table by a (?), which is meant to suggest that few words of the subject belong to the given category. No cases are twice counted in the totals.

SUMMARY VII.
Connection of Word-color with Letter-color.

(a)	(b)	(c)	(d)	(e)	Total.	No Letter-color.	Total.
? 9[4]	? 5[2]	? 4[2]	? 1[3]	? 8[5]	27	11	38

The value of our records, in explaining the phenomenon which they describe, must be admitted at the outset to be suggestive rather than demonstrative. Of possible theories there are, of course, two, which may be roughly characterized as the psychical and the physiological. The first refers

pseudo-chromesthesia and mental forms to ordinary associations, probably of childhood. The second finds no explanation, except in an assumed cerebral peculiarity—an especially close connection between certain brain tracts, especially the visual and the auditory. This assumption of the existence of apparently inexplicable brain peculiarities is, however, little more than a confession of ignorance; and the psychical theory will evidently be preferred, if it can be substantiated. But it is difficult to draw positive conclusions from the assertion or from the denial by adult subjects of such possible explanations; for, on the one hand, it is entirely probable that many actual associations are forgotten, while conversely, it is possible that plausible explanations are imagined and then assumed in good faith as the actual ones. It is at least certain, as the following table shows, that almost all colorassociations and forms date back to childhood.

SUMMARY VIII.
Beginning of Pseudo-chromesthesia and of Forms.

	In CHILDHOOD.		Later.		Time?	Вотн²	Тотат	Unv.	Total Cases
	Surely.	şı	Surely.	91					CASES
Pseudo-chromesthesia began Forms began	29 48	1 7	1 3	3	12 21	2 3	45 85	0 0	45 85

In the case of pseudo-chromesthesia, the number of explanations actually offered are recorded in

	Explanation		Total	No Explan	Unv.	Total Cases
	Sure.	?	TOTAL	TO DAI HAM.	0.111	TOTAL CASES
Color with words (including some letter and music-associations). Color with words only. Color with letters only.	3	10 4 1	13 7 1	12 3 1	2 1	27 11 2
Color with music only.	3		3	2		5
Totals.	9	15	24	18	3	45

¹ Doubtful.

² Part of the color-associations (or forms) in childhood; part later.

This result is less significant than it appears to be. greater number of explanations are, as is shown, doubtful or "When I was three years partial, including such as these: old, I had a playmate named Ethel, who had the loveliest blue eyes I have ever seen. She made such an impression upon me, that now the word Ethel seems almost a synonym for blue:" and this other: "'Harry' may be 'yellow' because associated with an imaginary Harry, with yellow curls." Color-associations with music seem most easy to explain and are in great number accounted for through a sort of emotional middle term; the connected color and sound are those which are apt to occasion similar emotions. A typical instance of this emotional connection between music and color is this "Color-hearing of musical sounds is due almost entirely to emotion. When I hear that which produces a pleasurable emotion, I immediately prolong this by seeing those colors which would produce the same emotion." "The tone of a violin," another says, "is very pleasurable, and blue is my color for my happiest moments." The explanation is still more detailed in the following quotation, which, however, suggests rather a deliberate connection: "As one naturally translates a lovely thought of one language into another, so the beauty of music was expressed in color I am naturally fond of red, which forms the foundation of my musical association, intensified into black and etherealized into pink; mixed with blue, for the passionate purple, and bothered with vellow to make the sullen and bitter discord of vermilion." The exact figures are these:-

SUMMARY X. Explanation of Musical Color-associations.

EMOT'L ASS'N.	Ord'y Ass'n.	Вотн.	No Explan. Sure. ?		TOTAL REC.		Total Cases
9	1	4	7	1	22	1	23

It is significant, on the other hand, since color with letters (regarded as sounds) seems obviously a simpler and more primary sensational experience than color with words, that we have but one direct explanation, and that a doubtful one, of a letter-color. This is given by a person who connects & with yellow or red and & with green, and the suggestion is that the pronunciation, spelling and meaning of the words

green, red and yellow may be the cause of the color-association. Now, most instances of pseudo-chromesthesia include both letter-color and word-color, and in half of these the word-color is at least partly explained by some ordinary association. It is not unlikely, therefore, that the letter-color is often the secondary experience and that letters take their colors from representative words. The figures are these:—

SUMMARY XI.

Connection of Letter-color with Word-color.

LETTERS ARE	CONNECTED V	WITH WORDS.	Unconni Wor	T	Unv	Total Cases.	
LETTERS ARE	Explained words.	Unexplained words.	? Explained	Unexplained	i	URV.	CASES.
	13	11	1	1	26	2	28

The lack of uniformity already noticed in the color-associations of different subjects with the same letters, implies, of course, the individual nature of the association and may point toward this same explanation of letter-colors through word-colors. The theory of Mr. Stevens¹ best suggests the possible connection between these colors and the childhood experience. He supposes that the color of each letter may be that of an object of whose name this letter is the initial. It is undoubtedly true that to all children the familiar horse, dog or bird is the typical one. If the sound of the word "dog" first suggests to an imaginative child his father's Irish setter, then the sound may be closely connected with the color brown; when the letters are learned, d stands for dog and takes on the color of the "dog par excellence." It must be remarked that this theory, ingenious as it is, still accounts with difficulty for some of the phenomena, for instance, for the vivid colors, red, green and yellow of so rare a letter as q; or for the red, black and vellow of z.

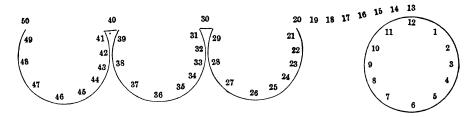
Definite explanation of forms, by identification with the shapes of familiar objects, occurs rarely in our records. Including even doubtful cases, only about twelve per cent. of our forms are explained.

¹ Pop. Sci. Mo. March, 1892.

SUMMARY XII.
Explanation of Forms.

	EXPLANATION		Tomat	No Explanation.	Hwy	Total Cases.	
	Sure.	91	TOTAL.	NO EXPLANATION.	ONV.	TOTAL CASES.	
Month-forms. Number-forms. Day-forms. Century-forms.	6 7 1 1	5 4 2	11 11 3 1	57 49 44 10	5 7 3	73 67 50 12	
	15	11	26	160	15	202	

Yet most of these explanations seem more obvious and satisfactory than those offered in cases of pseudo-chromesthesia. The following, for instance, has an interesting history:



The subject says: "I cannot explain the origin of the almost straight lines between 12 and 20, but the curves came from the fact that I learned to tell time before I learned to count, and when I did learn, everything reverted to the picture of that old clock."

It is evident, however, that if the psychical theory were supported merely by the fact of these remembered associations, it could claim little value. But the natural childhood associations which it hypothesizes may certainly have existed, though they are now forgotten. An observation of the forms themselves shows that they are chiefly those of ordinary objects, always frequent and sometimes prominent in the child's environment. The classification is the following:—

¹ Doubtful.

SUMMARY XIII. Nature of Number-forms.

SIMPLE	Broken	Lines.	Curvei	Lines.	Total Rec.	UNDEC	Тотат	
STRAIGHT LINES.	Simple,	Complex.	Simple.	Complex.		ONREC.	l'Olab.	
17	15	16	7	5	60	7	67	

Of Month, Day and Century-forms.1

and the second s	STRAIGHT	TRAIGHT LINES. CIRCULAR. RECTANGU			Total Rec.	Thypma	Tomir		
	Straight	Broken	Circles	Curved	Squares	Rectangles		ONREC.	TOTAL
Month- form.	7	3	23	22	7	6	68	5	73
Day- form.	21	7	1	8		10	47	3	50
Century- form.	4	3	_	2	1	1	11	1	12
Total.	32	13	24	32	8	17	126	9	135

A classification of the points at which number-forms bend also bears on this probable explanation by showing that three-fourths of these turns are at numbers which are prominent in early arithmetical exercises and in ordinary usage.

SUMMARY XIV.

Turns of Number-forms.

At 5 and its multiples,	1
At 10, and at 10 and multiples,	17
At 12; 8 and 12; and 10 (and 10 with multiples),	8
At 20; 20 and multiples of 10,	5
At 100 and multiples,	1
Combinations of these forms,	5
Turns at each number,	1
Irregular turns,	8
Total,	46
No turns,	13
Total recorded,	59
Unrecorded,	9
Total,	68

¹For examples of these forms, Cf. Figs. 1, 2, 4, Plate I.

Among the eight under the head "irregular turns," occur some so peculiar in form as to defy all identification with childhood objects, yet even these include many bends at the familiar 10's and 12's; and it is not unlikely that turns at such numbers as 6, 8, 15, 19 and 27 might be explained by some fact of great importance to the childish mind, by the consuming admiration for a playmate who is eight years old, or by the impressive death of some one at twenty-seven.

The argument from utility, which treats the "form" as a survival of a useful device, may finally be emphasized in support of the psychical theory. The visualization of numerals or of word-series may be an important aid to memory, especially in a child's first struggle with numbers. dental associations of this sort may then be perpetuated because of their helpfulness. This genesis of forms is explicitly recognized by one subject, who says, "the other forms have arisen simply from the ways that I have used to remember." There are many more emphatic assertions of the usefulness "I cannot realize," one man writes, "how any one can dispense with something of the kind. It seems to me that without this form, numbers would have no meaning, and a person would be entirely lost in considering them." "In the study of history," a student says, "I always associate events or men with the century which stands in such a place on the form. For instance, Queen Elizabeth reigned in the middle of the fifteenth century, which stands thus

"When I add numbers," writes another, "I invariably think of my form, e. g., if I add 5 to 27 . . . I feel as if I passed on a step. When I multiply, I feel as if I jumped, as it were, from one place to another, e. g., 6x9 is a long jump compared with 3x6." Even the musical memory may be aided. "If I hear an opera," one subject says, "I can come home and almost play it by colors; I know what chords make a certain combination of colors."

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The rare occurrence of distinct alphabet-forms may be explained on this principle as due to the fact that "the child needs no mnemonic system by which to learn the letters. He already sees them before him on blocks or in picture-books. Moreover, the alphabetic sequence is not so important as the numerical sequence."

¹Some such irregular forms are shown in Figs. 5 and 6, Plate I.

²Cf. G. W. T. Patrick, *Pop. Sci. Mo.* Feb. 1893. ³ Extract from an essay by Blanche L. Clay.

The following table shows the relative number of those that are sure that they have been helped by the possession of forms:—

SUMMARY XV.
Utility of Forms.

	UTILI	r¥.		No Utili	TY.	TY	Tomas Grand
	Sure.	?	lot.	Sure.	?	Unv.	Total Cases.
Usefulness of month, day and century-forms, in remembering dates and appointments.	27	4	31	39	5	3	78¹
Usefulness of number-forms both in remembering dates, etc., and in mathematical operations.	29	5	34	21	5	7	67

Whether or not one admits, as I have been arguing, that the forms which correspond with familiar shapes and those which are of acknowledged use to the possessor, as well as those which admit of definite explanation, are likely to be due to psychical associations,—it is yet worth while to observe that all these forms may be classified under one, at least, of these classes, while most belong to more than one.

The helpfulness of pseudo-chromesthesia is less obvious, yet it is reported in about one-fourth our cases. "Colors do not look right," one subject writes, "unless a word is spelled right. For instance, I spelled permanent, the other day, with two a's, and it did not look pale enough." Another is helped in writing rhymes, and a third in committing music to memory.

The pleasurableness of the experience also is very general, and may be a reason for its perpetuation. In several instances, already quoted, the favorite color has been definitely recognized as the basis of all the color-associations. The positive pleasure of this color-experience is naturally greater, as appears from the last part of the following table, than that derived from the usually simple and pre-eminently useful "forms."

¹The number refers to subjects, not to different forms.

² In the cases in which number-forms are used as year-forms or as day-of-the-month-forms.

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SUMMARY XVI.
Psychical Value of Pseudo-chromesthesia.

		YE	S.	No	•	Neither	Вотн	Тотат	IIvv	Total
	Sure	?	Total	Sure	?	Nammar	Times		OAV.	
1. The subject's memory is helped.	10	3	13	30		1		44	1	45
2. The subject is fond of the associated colors.	13	8	21	12	2	8		43	2	45
3. The subject finds pleasure in the experience.										
(a). Of pseudo-chromes- thesia.	14	11	25		1	13	4	43	2	45
(b). Of forms.	24	9	33	1	5	46		85		85

The diminution of both phenomena is slighter in our subjects than is usually supposed, perhaps because of their comparative youth; it suggests the effect of lack of attention when the experience has become monotonous and has outlived its usefulness.

SUMMARY XVII.
Relative Increase and Decrease

	Increase.		ASE. DECREASE.			Neither	9	Total	Unv.	Total.
	Sure.	?	Sure.	?						
Of pseudo-chromesthesia	4	2	9	3		18	7	43	2	45
Of forms.	14	5	6	6	1	39	14	85		85

The hereditary tendency of colored-hearing and of forms indicates, of course, the importance of the accompanying cerebral changes. Including doubtful ones, the number of negative cases is probably far too high, is ince in many cases one person has answered for an entire family, while, in almost all, there has been no opportunity for careful investigation.

¹The cases under "No?" are merely those in which the subject has no knowledge that members of his family have forms, etc.

SUMMARY XVIII.

Hereditary Tendency.

Do members of a family have

a. Pseudo-chromesthesia?									b. Forms?						
	YES	Yes. No.		CASES		CASES	YE	No.				TOTAL CASES			
Answers.	Sure	?	Sure	?	TOTAL	UNV.	TOTAL	Sure	?	Sure	?	TOTAL	UNV.	TOTAL	
1. Of subjects with pseudo-chro. only.	21		9	10	21	1	22	0		8	12	20	1	21	
Of subjects with forms only.	5		14	42	61	1	62	24	24	6	7	61		61	
3. Of subjects with both.	5	1	4	13	23		23	7		4	13	24		24	

The remaining tables embody the answers to questions of less importance, yet of a certain interest. It is not surprising to find that nearly all our subjects are good visualizers, but it is more remarkable that nearly all of them draw or paint.²

SUMMARY XIX.

Colors with Odors, Tastes and Touches.

Visual Imagination.3

	YES	3.				CASES	YES	3.	No	.	,		CASES
	Sure	?	No.	TOTAL.	UNV.	TOTAL	Sure	?	Sure	?	TOTAL.	UNV.	TOTAL
Subjects with pseudo-chro. only.		1	17	18	3	21	11	7		1	19	2	21
Subjects with forms only.	1	2	58	61		61	52	5	3	1	61		61
Subjects with both.	1	3	20	24		24	23			1	24		24

¹Color-association with odors.

²This question was asked by MM. Beaunis and Binet of their subjects. Cf. Revue Philosophique, April, 1892.

³See note 3 on page 454.

SUMMARY XIX.—Continued.

Drawing or Painting.3

Dramatization.4

	Yı	es.		,		CASES	YES	ı.	No		.;		CASES
	Sure.	A little.	No.	TOTAL.	UNV.	TOTAL	Sure	9	Sure	?	Toral.	UNV.	Total
Subjects with pseudo-chro. only.	5	4	10	19	2	21							
Subjects with forms only.	16	11	34	61		61	7	4	72	1	84	1	85
Subjects with both.	7	11	6	24		24							

Under the head "Dramatization" are included some simple cases in which, for instance, certain numbers or days are essentially disagreeable or "sharp and keen." There are, besides, more elaborate personifications, like the following:—

"T's are generally crabbed, ungenerous creatures. U is a soulless sort of thing. 4 is honest, but mathematically angular and ungraceful. 3 I cannot trust, though it is fairly good-looking in personal appearance. 1 is dark in complexion. 9 is dark, a gentleman, tall and graceful, but politic under his suavity."

"For numbers, I entertain either a like or a dislike; for instance, 11, 13 and 17 are especially disliked, I suppose because they are prime. My feeling for 11 is almost one of pity."

"The letters are very individual, for instance, K seems like a young woman, a friend of L, which seems like a daughter to M. N seems to be a sort of maiden aunt, sister to M. O is a young man connected with M as a nephew. He connects M and N with P, an older friend of his. Q is odd and stands by himself as rather an eccentric middle-aged man. R is like a maiden lady, an advisory friend of S, a young, handsome girl. T is the devoted admirer of S."

To one person, written capital letters have different expressions according as they are made in different ways, for

³In the investigation of forms, both these questions were divided, with the following result: One-fourth of those with good visual imagination of form do not strongly visualize color; and about one-ninth of those who draw do not also paint.

⁴I am inclined to think that this summary over-states the negative, for those who answered the question in writing may have misunderstood it. The question was not asked of subjects of pseudo-chromesthesia.

example, one style of written I is "honest and well-intentioned, but dictatorial and overbearing," while, written in another fashion, I is "a crabbed old miser." The subject adds: "This feeling is so strong that at times, when I have wished to produce a certain impression by a certain sentence, I have noticed that the capital letter at the beginning looked so at variance with the tone of the sentence that I erased it and made it in a different way, in order to keep the harmony."

The results of this paper, so far as they are not merely a summary of statistical observation, may be briefly summarized: We have no direct proof for the psychical theory of forms or of pseudo-chromesthesia. An examination of the different forms shows us, however, that most of them may be plausibly explained by the hypothesis of forgotten childhood-associations; and this probability is increased by the fact that such associations would be useful in learning the number series and in remembering dates.

Musical color-hearing and some name-associations are explained in the same way. Color with the letters may also be accounted for by arbitrary and forgotten childhood-associations; but it is possible that the explanation in this case is primarily a cerebral one. In general, however, such color-associations are either useful or pleasant, so that, even if their occasion be cerebral, their continuance, both in the individual and in the family, is largely due to attention and to cultivation. ¹

APPENDIX.

The descriptions, which follow, of special cases are extracts (except the final one) from essays by different students. The last two are records of self-observation; the others embody the results of personal investigation.

PSEUDO-CHROMESTHESIA.

CASE A.

Miss A. is a girl about nineteen years old, who says that she has had this experience ever since she can remember, but that it has never occurred to her as anything unusual. She sees the color only when she hears the letter or word, that is, when someone else speaks it; when reading, unless she stops to say the word to herself, she has no impression of color.

The phenomenon manifests itself with her, especially with the letters, both vowels and consonants, and with words only in so far as the initial letter throws the color over the rest. For example, as a is blue, Alice is blue, and because s is yellow, Sunday is yellow. The same rule holds good of figures; 2348 would be red, because 2

¹I am indebted for many suggestions in the collection of statistics and in the discussion of theories to my friend and former student, Miss Helena M. Corey.

is red. There are, however, some exceptions. When there are two consonants at the beginning of a word, their colors frequently blend, producing a color which is a combination of the two, for instance, in the word Thursday, the color is a red-brown, a combination from t which is red, and h which is brown. Moreover, there is a strange exception in the name Monday, which appears blue, while m is a decided red. Miss A. says that she cannot account for this, unless the association of blue with Monday in "blue Monday" replaced the original color red, and became more firmly fixed in her mind than the color produced by m, the initial letter.

There seems to be no especial arrangement of the letters according to color, although the same color is often repeated in the course of the alphabet. For example, d, j, n are all brown, whereas f, k, w are steel-gray. Each letter has a distinct color of its own, apparently without reference to form or sound. But it sometimes happens that letters with somewhat the same form have the same color, for instance, both o and c are white, and the figure zero is white also, while the figure 1 and the letter i are both black, but 3 and 6 are black also, and they differ so totally in form that the rule will not

hold good in their case.

To illustrate further that the sound of the letter has no influence, I asked Miss A. about the colors of "ought" and "aught," and she said that "ought" is white, while "aught" is blue, following the rule of the initial letter.

The colors corresponding with the letters and numerals are as

follows:-

a, b, and 8, blue. q, s, v, y, 4, 7, yellow.

Most of the colors, it will be observed, are of the softer, more Some of them, however, are more distinct than others, for example, the red, blue and black, of which the red is by far the most vivid, apart from its being the brightest color.

But the more subdued colors, especially those in the écru shades, are almost indescribable. They are so vague and indefinite that it

is hard to give them any name.

Miss A. does not usually see the letters or words themselves colored; she has merely a vaguely outlined image in the characteristic color, when the word is spoken, for instance, with the word "other," she sees a sort of "streak" of white. But the very vivid colors affect the letters themselves, for example, m and t always seem red

The subject has no explanation to offer. She thinks, however, that her colored-hearing cannot be the result of early association, because the colors have been produced only gradually, and have increased with time instead of fading, as might more naturally be the case if this were a mere childhood-association.

MARY L. SMITH.

CASE B.

Miss B. is also about nineteen years old. She has had coloredhearing ever since she was a child, long before she could either read or write. With her, however, colors were first associated with names and only later, since she has thought about the matter, has she associated colors with the letters of the alphabet. B.'s mother first noticed the peculiarity, when she was asked to suggest names for the marbles with which the child was playing. The mother proposed names, but B. rejected most of them as unsuitable, because, she said, they were not of the same color as the marbles. This incident impressed the mother as very curious, for she does not know that any others of the family have colored-hearing. She had never, however, spoken of it particularly until B., who did not remember the experience, began talking about colored-hearing, a few weeks ago.

Miss B.'s colors, for the letters, are these:—

a, blue.
e, yellow.
i, dark red.
o, light gray.
u, scarlet.

p, dark blue. b, black. c, yellow. d, dark red. q, dark red. \bar{r} , brown. f, yellowish-brown. g, brown. s, dark blue, almost black. g, bro... h, cream. t, still darker blue. v, dark navy-blue. j, brown. w, red. k, dark blue. x, green. y, cream-color. l, red. m, brown. z, dark, almost black. n, deeper brown.

Miss B. sees colors with all letters and with almost all words, but the association is most marked with proper names. The following are her "month-colors":—

January, red.
February, straw-color.
March, blue.
April, purple.
May, gray.
June, red.

July, deeper red. August, sky-blue. September, brownish-yellow. October, light yellow. November, bluish-gray. December, reddish-brown.

The word may be spoken or written; but the color does not always appear with an imagined word. No other sounds of the human voice or of musical instruments ever suggest color. There seems to be no rule, such as that of the initial letter, for the association of colors with words, but each separate word and letter has its distinct color, and if the colors seem to be duplicated there is a decided difference in the shade. For instance, Sarah and Stella, which have the same initial letter, are totally different in color, the former a bright blue, and the latter corn-color. Harry, which has almost the same sound and in great degree the same letters as Carrie, is dark red, while Carrie is a very dark blue, almost black. If we compare these with the colors of the letters, we still get no explanation of the coloring.

CASE C.

Miss C.'s colors for vowels are as follows:

a, dull gray.
i, nearly black, very dark green.
o, nearly colorless.
e, dull red.

u, dark bottle-green.

Her colors for consonants are these:-

```
b, brown.
```

c, vivid lemon-yellow, the brightest letter of all.

d, dark brown.

f, ruddy brown.

g', purple. h, greenish-yellow.

j and k, no color. l, dull gray.

m, brown, especially distinct.

n, nut-brown.

p, lead-color (tint of purple).

q, dark green.

r, deep brown.

s, white with brown spots; (in combination) pink.

t, bright green, a very clear color.

v, fawn-color.

w, heliotrope.

x, black.

y, mixture of lavender and blue, Dutch-blue.

z, yellowish-drab.

It is noticeable that many of Miss C.'s colors are of a dark shade; she says that the colors were much more distinct in childhood and it is possible that they have lost something of their original brightness. Upon being asked whether the impression of color comes when the word is merely heard, or when read and imagined, Miss C. replied that it comes when imagined and when heard, never when the printed page in black and white is before her, unless she stops to imagine the word or letter. The black and white seem to dispel the colors. The colored spoken letters do not seem quite so distinct as those which she imagines. No other sounds than those of letters and of words produce any color, and the quality of the voice does not make the slightest difference in the colors, which are the same in connection with all voices. Her own moods have no effect on the colors.

The separate letters of a word are all themselves colored and on a rather dark background. In most cases the color comes from the first letter, but very often the word is shaded; this effect does not usually come from combination, but each letter is seen as a separate one, in its own color. These statements have been verified. The word "Carrie" appears to Miss C. a bright yellow, influenced no doubt by the c, which is lemon. "Harry" is greenish-yellow, the exact shade of h. Here the similar sound of the words (irrespective of the initial letters) seems to have no influence. "Helen" is dark green, influenced by h and perhaps somewhat by l, a dull gray; this seems like a combination of the two colors. "Stella" is one of the few shaded words, a very delicate pink, shading into a dull green. I am sure that this comes from the combination, for s (white with brown spots, when isolated) is invariably a very dull or delicate pink when with other letters; e is a bright green, and in combination with l and a, dull gray, would very likely give dull green.

The subject occasionally finds that in trying to think of a word, the flash of color comes just before the word, though, until questioned, she had never thought of the significance of this experience as an aid to the memory. These flashes, however, occur very seldom. The word-color has existed ever since she can remember; in childhood, it was often a source of amusement and she never doubted that other people associated colors with words and letters. She is the only one in her immediate family who has colored-hearing; until her attention was called this spring to the peculiarity, she had never mentioned it at home, and her family were much surprised at her experience.

Miss C. can offer no theory of colored-hearing, but the fact of having learned one's letters from blocks is not a satisfactory explanation, since it would account for only a few colors, which should then be often repeated with the different letters.

AGNES M. SHAW.

CASE D.

The subject is a girl about nineteen years old, who has always been in the habit of seeing colors in connection with all letters of the alphabet, including Greek letters, with words made up of these letters and with figures. The complete list of the associations follows:—

a, light brown. l, blue-black. e, reddish-yellow. m, brown. i, black. n, gray. p, dark blue. o, white. u, cloudy white. q, blue-black. r, grayish-white. b, dark brown. s, reddish-yellow. t, blue. d, white. d, dark blue. f, brown (F gray). v, dirty white. g, dark blue.
h, brown.
j, black. w, brown. x, red. y, grayish-white. k, light blue. z, red.

The words and consonants seem to be about alike in clearness, but the capital letters, besides being larger, are more distinct and appear brighter and more conspicuous than the others; sometimes they even assume different colors or shades, as in the case of small f, which is brown, while capital F is gray, or, as in the case of small f and capital f0, which are different shades of blue. There is also a marked difference in shade between written and printed letters, so that f1, f1 and f2 have different shades. Miss f3. thinks that the distinction is due to their difference in shape.

There seems, however, to be no difference in shade, according as the letters are spoken or written. If a word is spoken quickly, it immediately assumes the color of the initial letter, but if the word is imagined, or slowly repeated or read, then not only the initial letter, but all the letters assume colors, so that a printed page seems to be illuminated; the letters are not on a colored background, but each stands out for itself in its own individual color. (Therefore, if, in an illuminated text, letters thought of by the subject as, perhaps, brown and white, are made, for instance, red and black, they do not appear natural, and must either be re-colored or be printed

in ordinary form to suit her taste.)

The subject has no explanation or theory of her colored hearing. The experience is not hereditary and is not connected in any way with childish associations of which she is conscious. She does not think that the colors can be closely connected with the sounds of the letters, for two letters sounding precisely alike are represented by very different colors, and two words pronounced alike, but spelled differently (as aught and ought or air and heir), appear very dissimilar when seen in colors. She thinks the color more likely to be associated with the form of the letters; yet one can detect no similarity of form in a, b and m, which are shades of brown; or in d, k, and p, which assume the various shades of blue.

¹There is, however, such similarity between b, f, h or between m and w (brown); and between g, p, q (blue).

Miss D. has a use for her colored hearing. She says that the practice of associating colors with letters aids her greatly in correct spelling and in committing words to memory.

MARY R. EASTMAN.

CASE E.

The subject is eleven years old and has had colored hearing ever since she can remember. All the letters of the alphabet, the names of days of the week and of months of the year, numbers and many Christian names, but no common names, are colored. The letters, either separately or occurring in words, are not themselves colored, and the image of the color is vague in form. Both sound and sight, but the sound more clearly, suggest color.

```
The list of letters with their colors is this:—
                           a, black.
                           e, grayish-white.
                           i, gray.
                           o, white (more of gray).
                           u, gray.
b, blue.
                                        p, red.
c, black.
                                        q, gray
                                        \bar{r}, reddish-brown.
d, blue (a little different from b).
f, dark red or black,
                                        s, bluish-black.
g, grayish-black. h, blue.
                                        t, brown.
                                        v, gray
j, bluish-black (more black). k, red.
                                        w, reddish.
                                        x, black.
                                        y, brown.
l, fast black.
m, a blue-and-red.
                                        z, black.
n, a blue-and-red.
  The colors of names of days are:—
Sunday, black.
                                         Thursday, whitish-gray.
Monday, reddish-brown.
                                         Friday, blue.
Tuesday, blue.
                                         Saturday, whitish-gray.
Wednesday, whitish-gray.
  The month-colors are:—
January, grayish-white.
                                         July, black.
February, reddish-brown.
March, black.
                                        August, grayish-white.
September, reddish-brown.
April, dark brown.
                                         October, dark blue.
May, pink.
June, pinkish.
                                         November, dull brown. December, blackish.
```

The colors with numbers are:-

1, grayish.
1, gray.
2, brown.
3, very dark blue.
4, light.
5, reddish-brown.
6, black.
7, brownish.
8, blue.
9, black.
10, reddish-brown.
11, gray-white.

12, gray-white.
13, dark-blue.
14, yellowish.
15, reddish-brown.
16, black.
Etc.
21, brown and gray (with colors distinct).
Etc.
30, dark blue (but lighter than

Etc.

It will be observed that 0 by itself is gray, but, occurring with the other numbers, unites with the color of the accompanying number,

making it lighter.

My subject paints well in water-colors, and is a fairly good visualizer. I have tried her several times, unexpectedly always, and she has invariably given the same color for the same sound, though the colors are often difficult to describe. They are frequently modified shades or a mixture of several colors. Indeed, in almost all cases of colored-hearing, it seems to be impossible for the subject to indicate the exact shade which a given sound brings to her mind. The color is, for instance, reddish-brown or grayish white more often than it is clear red, brown, gray or white. If the colors, as she describes them, are put upon paper, they never quite satisfy her.

My subject can not account for the phenomena of colored-hearing in her own case. She says that she "feels queer" if the colors do not come as soon as she hears the words corresponding with them. I can not myself determine any rule which governs her color-associations. The initial letter does not often color the whole word: Wednesday is not the color of W, nor Oscar of O. Neither do the vowels give coloring to the word, and the words do not seem to be the result of the mixing of the various colors of its component letters. But rhyming words, for instance, Harry and Carrie, frequently, though not not always, suggest the same color, indicating again that the color is directly connected with the sound.

BLANCHE L. CLAY.

NUMBER-FORMS.

CASE F.

(The illustration which accompanies this description is from a stereoscopic photograph of a twelve-foot wire, bent by the subject into the characteristic form. The various rests and cords which complicate the picture were necessary to hold the wire in place. The form is represented by the heavy line. The accompanying

description is written, as appears, by the subject herself.)

I have had my number-form since early childhood. I can not remember when it began, neither can I remember when and how I learned to count. I have a dim recollection of being set to learn the addition table and of making use of the form, which was even then in existence. It occupies a subordinate place in consciousness, and, though always present when the subject of thought has to do with figures, it may not be distinctly imaged. For instance, I always make use of my number-form in solving a problem, because it helps me, but my mind is closely occupied with the problem and is only dimly conscious of the form.

The numbers, which run in a general north and south direction, are not all in the same plane, and my position with respect to them frequently changes. I usually seem to stand just outside the line of numbers, and near, but never on the number most prominent in my mind at the time. I feel much more at home among the numbers about 20 than in any other place on the form. Generally speaking, the plane of the number rises from 1 upwards, but there are many small ups and downs in the line. From 1 to 5 the course is downward; at 6 comes a sharp and upward turn; 8 is lower than 6 and 7 and is in a corner. From 8 to 12 the course is upward, with a bend at 10. At 12 comes another sharp turn; from 12 to 19 the line descends, making a sudden descent at 16, and a steep ascent at 19.

¹ Fig. 1, Plate II.

The line bends slightly at every number between 15 and 19; 18 is sunk down in a corner, a little like 8. 20 is higher than 19, and at 20 comes another bend. 21 is on a level with 20. The numbers 21-29, 31-39, 41-49, etc., follow the same order as those from 1 to 9. is higher than 20, and may be seen from the latter number, since the intervening numbers are in a lower plane. 40 is lower than 30, and is at its right. 50 is higher than 40, and 60 is higher than 50, but lower than 30, which is the highest number in the line. Both 50 and 60 are to the right of 30. At 60 there is a turn to the left. 70 is lower than 60, and 80 is lower than 70. At 80 there is a southward bend. 90 is as high as 60, and is situated in a bend in the line. 100, which occurs in another bend, stands east of 90, and is almost

as high as 30.

Outside the form on all sides is darkness; the line itself grows dusky at 70, and beyond 100 there is total darkness, except when that part of the line is directly fixated. The numbers between 100 and 200 run in the same order as those between 1 and 100. true also of the numbers between 200 and 300, 300 and 400, etc. The numbers 100, 200, etc., up to 1000, that is, the even hundreds, follow the same order as the numbers between 1 and 10. The same is true of the even thousands. 1,200 and 12,000, 1,500 and 15,000, are like 12 and 15 respectively. The numbers between 1,000 and 2,000 follow the same order as the numbers between 1 and 1,000, etc. Beyond the thousands the form grows dim and disappears. From 1 to 10, and from 20 upward, the numbers seem more or less in shadow, when not distinctly imagined, but between 10 and 20 there always seems to be a bright light. These latter numbers occupy a larger place than any other system of tens in the line, although the numbers between 1 and 10 are rather spread out. The 2's in the 20's and the 3's in the 30's, etc., help to fill up space and make the numbers seem more crowded together. In still higher numbers the hundreds and thousands take the attention to some degree from the figures in the units' and tens' places; thus in 178, I think of the 100 as much as of the 78. The 100 does not occupy the mind enough to get placed in any form, but it prevents my giving 78 its place in the line as quickly and as vividly as I should if it stood by itself, without the 100.

From one number in the form I can often see others at a distance. From 1 I can dimly see 10, and from 10 I can plainly see 20, 16, and a few others in the vicinity of 15; 13 is partially hidden from 10 by From 12, which stands almost as high as 30, I can indistinctly see the latter number and 40, though many of the intervening numbers are hidden from view. From 30 I can look across to 90

and 100, and even beyond.

Each number in the 'teens seems to occupy a comparatively large place in the form, which curves in passing from one number to the This is partially true of the numbers between 1 and 10. The order in the 'teens is unlike that in any other part of the form, except when the 'teens recur, as in 113, 213, etc. All the numbers

seem to be printed, and lying down on a dim background.

I associate character and sometimes sex, generally female, with numbers. 1, 2, 4, 7 and 8 are reliable, quiet, well-disposed, but not brilliant numbers. 3 is a sharp, shrewd, noisy and disagreeable number, always making as much trouble as possible. 5 is sprightly and merry, happy, and a number to be petted. 9 is dignified, though a little like 3. 10 is well-disposed and dignified. 11 is rather disagreeable, but not troublesome. 12 is a dignified, protecting number, capable of ruling all the lower numbers even 3 tecting number, capable of ruling all the lower numbers, even 3, and always treated with respect by the larger numbers. For 13 I

always have a great antipathy. It has all the disagreeable qualities of 3 added to a pertness and aggressiveness which make it repugnant to all the other numbers, with which it seems never to associate. I never wanted to be 13 years old. 14 is like 4. 15, although like 5, always seems strange, irregular, and out of place in the system. I frequently forget it in counting. I feel as if I had to go out of my way to bring it into the form at all. It seems entirely unmanageable. I always feel a great respect for 16. All these numbers are dignified and well-disposed, not brilliant. 18, however, is more important and occupies a larger space than 20.

The numbers in historical dates I think of in a way different from that in which I think of those in the ordinary number-form. The numbers which represent the century in which an event occurs are quite prominent in early dates, e. g., in the date 1020, I think scarcely at all of the 20, but the 10 seems to occupy the place occupied by 10 in the ordinary number-form; I do not think of it as 1000, or even as 10,00, but simply as 10. If I think of the date sometime, or in connection with other dates in the same century, my attention becomes fixed on the last two numbers, which take their ordinary places. The nearer the date approaches the present, the more I think of the last two numbers. The numbers in centuries previous to ours seem to occupy a smaller space than the corresponding numbers in our own century. 1867 seems comparatively near. 1767 does not occupy a space analogous to 1867, but seems to occupy one analogous to 1857, etc. I never think of historical dates in connection with 1000. It is never 1,867, but 18,67.

BLANCHE L. CLAY.

CASE G.

The bent wire is as good a material representation as I can give of the fact that successive numbers from 1 upward to 100 have a space relation to each other, such that if I consider the succession as a whole, I am conscious of mentally glancing along an even but more or less abruptly turning path, tending upwards at about 45°, but growing steeper among the upper numbers. In this glance over the whole series I locate myself more or less definitely in two positions: at a point about half way between 0 and the point on the base directly beneath the 30, for the early numbers, but for those above 25, at a point in the vicinity of 20 (a little in front of the place for 20, as seen in the cut); nevertheless, I reserve the right, as it were, to take other positions, for I find that I often consider limited portions, e.g., 70-80, from a nearer point of view. From 50 upwards the form is not as definite as below, and between 50 and $7\overline{5}$ it is not as definite as from 75 to 100. In counting beyond 100 a mental tally is kept of the hundreds, and the fractions are repeated along the form; the "mental tally of the hundreds" is quite closely associated with the printed figures 200, 300, and so on for higher numbers.

Multiples of 10 are naturally the most prominent positions along the form, and in a rough way the straight lengths between the bends include ten units, though at 30 there is hardly a perceptible turn, and the exact turning points are not at 10, 20, etc., but at 12, between 18 and 20, and between 40 and 42, thus suggesting an association with multiples of 6. Above 50, though not as definite, the turning points are at 60, 70, etc. The length also of the upper decades, as if foreshortened and distant, are less than of the lower.

¹ Fig. 2, Plate II., from a stereoscopic photograph.

[Owing to the position of the form relative to the camera, the bend at 19 does not appear as distinct as it is. The line from 20 to 40 is in a vertical plane nearly parallel to the vertical plane through the line 0 to 12.]

I can not explain the form from any associations of drawings, pictures, things or incidents. It may, however, have resulted

unconsciously from many such early associations.

While the form gives position to numbers, it does not have much to do with processes of treating numbers, except, perhaps, for adding, in which operation I follow the increasing sum along the form. 4×5 , however, does not mean that I shall take four lengths of five units each and measuring them off on the form, reach 20. 4×5 , 3×10 , etc., are mental relations whose equivalents, 20, 30, etc., are located on the form. Moreover, in written work with

figures very little reference is made to the form.

Other relations have forms in my mind: as dates, not at all connected with the number form, months of the year, days of the week; and as a rule my thoughts have a background of the mental pictures of places, things and people with which they are concerned. Dates, i. e., the years, seem to slope downward away from me if they are past, and upward if they are future. The months of the year form a closed cycle, in shape a quadrilateral, its corners between December and January, the middle or last of March, the end of June, and the middle of September. Undoubtedly this is connected with school terms and recesses as well as with the seasons. Days of the week succeed each other in a straight open series.

My use of such forms in thought seems not to be related with any special trait of character, for I do not draw or paint, am not extremely methodical, and do not have more than an ordinary memory for the forms of things; yet my taste is toward applied rather than pure mathematics, and I find that I tend to give a geometric interpretation to a mathematical expression.

This habit or trait of using space-forms for thought is sometimes helpful in getting conceptions and in remembering, often it is a

hindrance.

ARTHUR E. KENDRICK.

PLATE I.

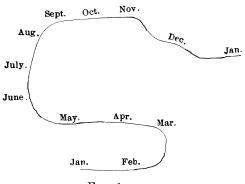
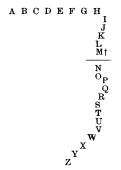


Fig. 1.

March. April.

* November * December * December * December back to the January position, for a fresh start.

Fig. 2.



 \dagger The line between M and N represents a fence between them.

Fig. 3.

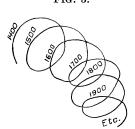
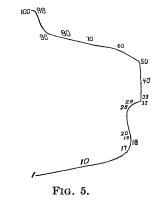


Fig. 4.



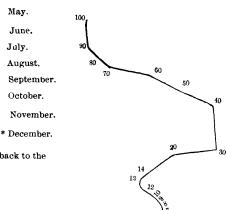
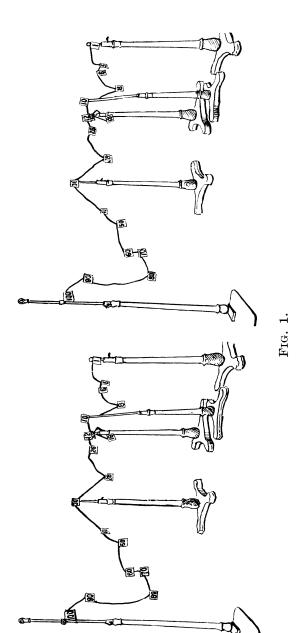
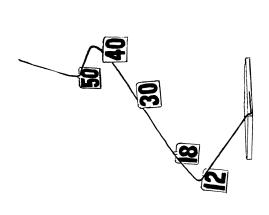


Fig. 6.

10	10 🗞	21	
2	<i>11</i> ∞	22	
3 🛆	12	23	\Box
4 🔀	/3 🔆	25	\$\bar{\alpha}{\alpha}
5	15-5	26	
6 🔀	16	28	
8	18 🞾	29	
9	19	30	8

29 is sometimes associated with other combinations of circles.





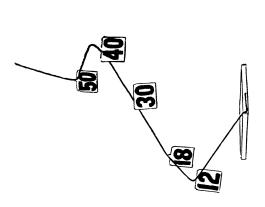


Fig. 2.